# GeoGebra is Golden

**Using GeoGebra to model the mathematics of the Common Core State Standards**

**Annual Meeting of the**
**North Carolina Council of Teachers of Mathematics**
**October 27, 2011**
**Greensboro, NC**

**Coordinated by**
**Leah P. McCoy**
**Wake Forest University**

# GeoGebra is Golden

**NCCTM, October 27, 2011**

**CCSS Mathematical Practices – *Leah McCoy***

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

**GeoGebra Activities**



**Complete activities: http://www.wfu.edu/education/ncctm11/**


Questions or Comments. *Leah McCoy*: mccoy@wfu.edu

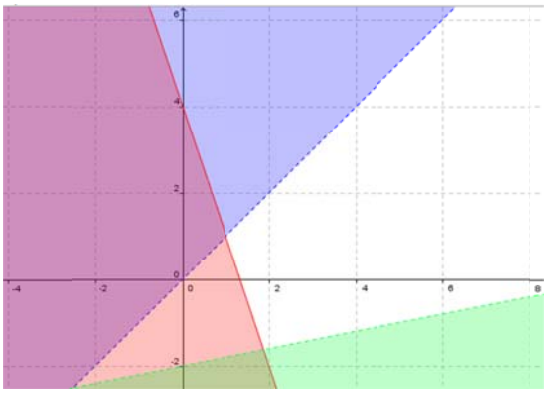
# Graph Linear Functions and Systems
# Graphing Systems of Inequalities

***Caroline Ewald***

CCSS: Algebra – Reasoning with Equations and Inequalities (A-REI 12)

CCSS: A-REI 12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

The newest version of Geogebra (4.0) allows you to graph inequalities. This feature is not included in earlier versions.



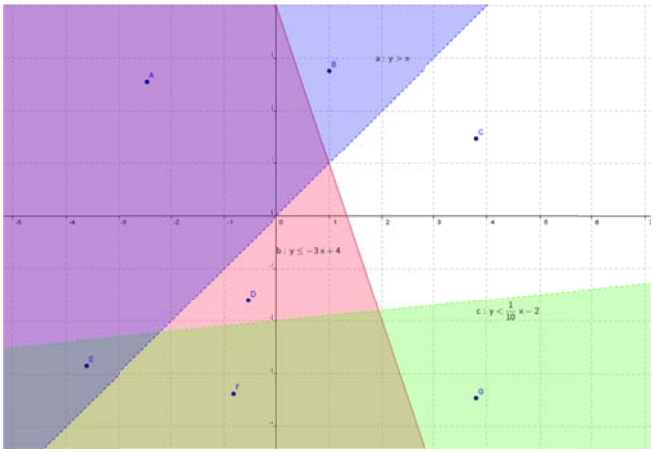
Construction:

1. Type the equations (in any form) into the input bar at the bottom of the Geogebra screen. You can use the "<" and ">" keys on the keyboard for strict inequalities, or >= or <= or choose ≤ or ≥ from the menu embedded within the input bar by clicking on the α button.
2. Type in the equations for the following system:
    a. y>x
    b. y≤-3x+4
    c. y< 1 10x-2
3. From the Algebra section of the Geogebra window, click on the equation for each of the equations you have graphed and drag the equation on to the graph to label the lines.
4. Change the colors of the second and third shaded regions: right-click on the shaded area and select "Object Properties". Choose the color tab and select a different color (red for the equation ii , and green for iii).
5. The 'Style' tab under Object Properties can also be used to change line thickness or to change from shading the chosen reason to "hatching" it. This option is effective for one inequality, but falls short for systems of inequalities, as the hatches coincide making it difficult to tell which areas are solutions for both inequalities rather than just one.
6. Using the toolbar at the top of the screen, add points to each section of the graph.

Investigation:

- Using the toolbar at the top of the screen, add points (A-G) to each section of the graph.
- Click 'Record to Spreadsheet' from the dropdown menu under the mouse tool.
    - In column A, type in the x-values of the points from your graph by typing in x(A), x(B), etc. *Make sure the letters of the points are capitalized!
    - In column B, type in the y-values of the points from your graph by typing in y(A), y(B), etc. Again, make sure letters representing points are capitalized.
    - In column C, check each point to see if it satisfies inequality a (y>x). To do this type in a(A), a(B), etc. The spreadsheet will give an output of true if the point satisfies the inequality and false if it does not.
    - In columns D and E, check each point to see if it satisfies inequality b and then inequality c.

Which points are solutions for this system of inequalities?



| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | -2.46 | 2.56 | true | true | false |
| 2 | 2.1 | 4.28 | true | false | false |
| 3 | 3.8 | 1.48 | false | false | false |
| 4 | -0.56 | -1.6 | false | true | false |
| 5 | -3.62 | -2.84 | true | true | true |
| 6 | -0.84 | -3.38 | false | true | true |
| 7 | 3.8 | -3.46 | false | false | true |
| 8 | | | | | |

# Graph Non-linear Functions
# Graphing Parabolas
***Anna Hester***

CCSS: Functions – Interpreting Functions (F-IF - 7)

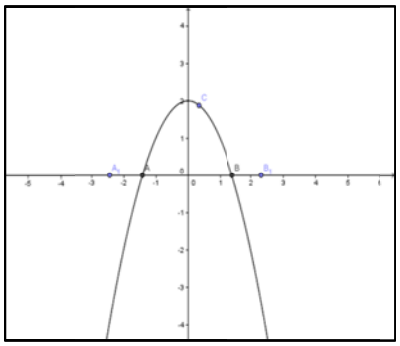
## GRAPHING PARABOLAS

CCSS F-IF 7a : Graph linear and quadratic functions and show intercepts, maxima and minima.

<u>PART 1</u>

CONSTRUCT

7. Type $f(x)=-x^2+2$ into the "Input."
8. Using the Line through Two Points Tool, create a line through two points on the x-axis.
9. Using the Intersect Two Objects Tool, intersect the parabola and the x-axis line
10. Re-label these points of intersection as A and B by right clicking on each point and choosing "Rename."



INVESTIGATE

d. What is the y-value of points A and B (found under "Dependent Objects")? What does this mean f(x) equals at these two points?

CONJECTURE

- Make a conjecture about how to find where a parabola intersects the x-axis.

CONSTRUCT

1. Using the Midpoint Tool, find the midpoint between A and B. Label it C.
2. Using the Perpendicular Line Tool, construct the line through C perpendicular to the x-axis.
3. Using the Intersect Two Objects Tool, intersect this line and the parabola. Label it D.

INVESTIGATE

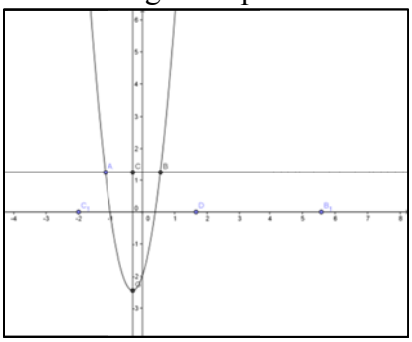
1. Are there any points with a larger y-value on the parabola than D? Smaller? (Use the "Point On Object" tool to move a point on the parabola)
2. Consider the part of the parabola to the right and left of the line through D, does there seem to be a relationship?

<u>PART 2: The form $ax^2+bx+c$</u>

CONSTRUCT (start a new window)

1. Type the equation $f(x)=5x^2 + 3x - 2$ into the "Input."
2. Using the New Point tool, draw a point on the parabola. Label it A.

3. Using the Line through Two Points Tool, create a line through two points on the x-axis.
4. Using the Parallel Line Tool, construct the line through A parallel to the x-axis.
5. Using the Intersect Two Objects tool, intersect the parallel line and the parabola. Rename B.
6. Using the Perpendicular Bisector tool, construct perpendicular line through AB.
7. Using the Intersect Two Objects tool, intersect this new line with the parabola. Label it D.



INVESTIGATE

1. Are there any points with a larger y-value on the parabola than D? Smaller?
2. Consider the part of the parabola to the right and left of this line, does there seem to be a relationship?
3. Calculate –b/2a using the coefficients of the equation. Where does this number show up on the dependent objects tab? What is the value of the equation when x= -b/2a?

CONJECTURE

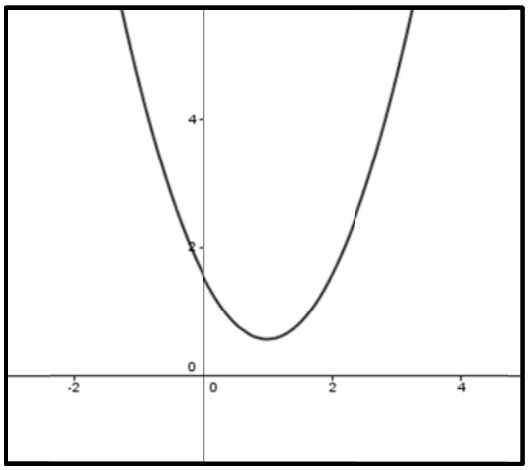
1. Make a conjecture about the relationship between –b/2a and the graph of a parabola.

# Identify Effects of Changes on Graphs
# Horizontal and Vertical Shifts

***Samantha Freiberg***

CCSS: Functions – Building Functions (F-BF - 3)

**Common Core State Standards – Building Functions**
**F-BF .3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.




**Construct:**

1. Use the Move Graphics View tool to center the origin in the window. Graph the function $f(x) = x^2$ by typing it into the Input bar at the bottom of the window.

**Explore:**

1. Use the Move tool to drag the parabola around the window. What do you notice about the equation for the function (listed under Free Objects on the left)? How does it change when you move the parabola?

**Investigate 1:**

1. Use the Slider tool to create sliders for $a$ and $b$. Set the Min. to -5 and the Max. to 5, with an increment of 0.1. Graph the quadratic function $f(x) = (x + a)^2 + b$. Drag the slider for $b$ first. What happens to the graph as you change the values for $b$?
2. Now drag the slider for $a$. What happens to the graph as you change the values for $a$?

**Conjecture:**

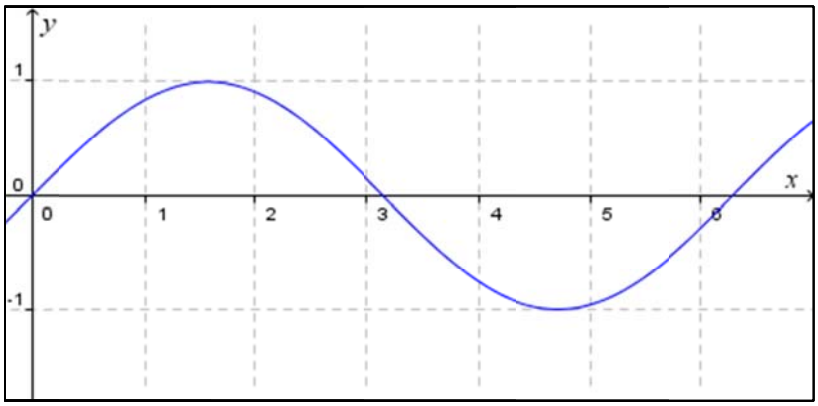
Write a conjecture for how $a$ and $b$ change the graph of $f(x)$ for $f(x + a) + b$. You may want to use the terms *horizontal shift* and *vertical shift*.

**Investigate 2:**

1. Test your conjecture with another function. Type *f(x)=abs(x+a)+b* into the Input bar to graph *f(x)* = $|x + a| + b$. Drag the sliders for *a* and *b*. What do you notice about the graph when you change these values? Does this confirm your above conjecture?
2. Now graph *f(x)* = $\sqrt{x + a} + b$ by typing *f(x)=sqrt(x+a)+b* into the Input bar. Drag the sliders for both *a* and *b*. Do your observations confirm your conjecture? Why?

# Model Trig Functions
# Sine Function Introduction
## *Tyler Claytor*
CCSS: Functions – Trigonometric Functions (F-TF 5)

F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.




**CONSTRUCT**

11. The general form of a sine function is Asin(Bx-C) + D. To display this in text on the screen, go to the "Insert Text" button and write this formula into the text box. The text will be small, but right click the text and select "Object Properties". In the second tab on this menu labeled "Text" you can change the font size from the default setting of "Small". This is located right next to the font.
12. Construct f(x) = sin(x) by entering the function into the Input field at the bottom of the screen.
13. Using the method from step 1, enter four more functions changing one variable from the general form at a time. You can use your own functions or use the examples below.
    a. g(x)= 2sin(x).
    b. h(x)= sin(2x).
    c. j(x)= sin(x-2).
    d. k(x)= sin(x) + 2.
14. Each function will be listed under "Free Objects" on the left hand side. To better visualize each function, right click each function's equation, select "Object Properties" and change the color of each function.
15. Clicking the circle to the left of each function shows or hides it on the screen. Hide g(x), h(x), j(x), and k(x).
16. Construct a point, A, on f(x). Every point you construct will be listed under "Dependent Objects" and their coordinates will be displayed.
17. Using the "Pointer" tool on the far left side of the toolbar drag the point on the function and observe the coordinates changing.

18. Construct a point, B, at the origin. The function will go up above the x-axis, intersect the x-axis, and then go below the x-axis. When it intersects the x-axis again, construct another point, C, at this intersection.
19. Construct line BC.

**INVESTIGATE**

e. As you drag point A across the function, what is the maximum y-coordinate on f(x)? This is the function's amplitude.
f. The function repeats itself across the graph. How long is line BC? This is the function's frequency.
g. The line that the function repeats itself around or oscillates around is the midline of the function. What is the midline of f(x)?

**CONSTRUCT**

One function at a time, display that function on the graph using the circles previously mentioned to left of each function and construct a point to drag on the function to observe the coordinates changing on the function.

**INVESTIGATE**

Answer the same three questions from the previous "Investigate" section for each function.

**CONJECTURE**

- Now that you have your "Investigate" answers for all five functions, compare your results for f(x) with the other four functions individually. Are there any differences that you see? If yes, explain.
- Using the circles previously mentioned to left of each function, visually compare f(x) with the other four functions individually. Are there other differences that you see not addressed in the first "Conjecture" question?
- Write an explanation for what each variable changes in the function.

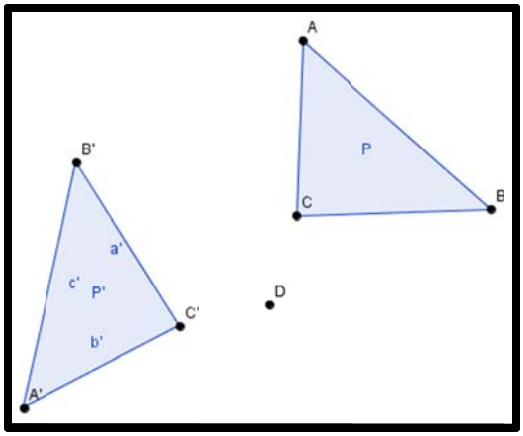
# Transformations

## *Sandhya Ghanta*

CCSS: Geometry – **Congruence (G-CO - 5)**

G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**CONSTRUCT 1 (ROTATION OF A GEOMETRIC FIGURE)**




1. Using the Polygon tool, draw a triangle. Right click on the triangle and rename the triangle P. Make sure to turn on the show label feature.
2. Selecting the New Point Tool create point D. Select the Rotate Object around Point by Angle. Choose triangle P, point D, and insert angle measure 120° clockwise.
3. Label the rotation triangle P'. Make sure to turn on the show label feature.

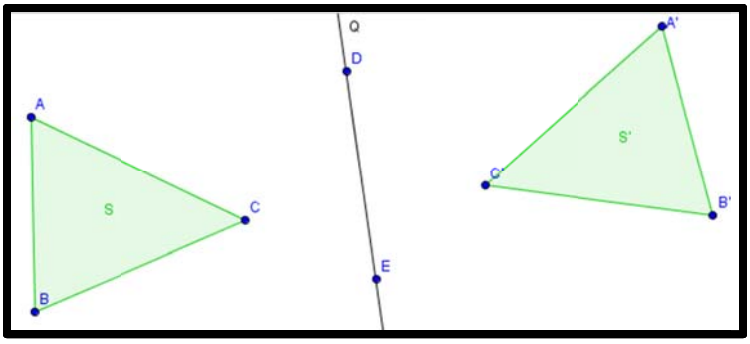
**INVESTIGATE 1**

1. Measure the lengths of segments AB, AC, BC, A'B', A'C', B',C'. Click on each segment, length will appear highlighted on left. For lengths to appear on the image select the Distance or Length tool and select the segments.
2. Select the Angle tool. Measure angles ABC, BCA, CAB, A'B'C', B'C'A, C'A'B'. Select triangle P and P' to have angles displayed on the triangles.
3. Name the sides of triangle P that are congruent to the sides of triangle P'.
4. Name the angles of triangle P that are congruent to the angles of triangle P'.
5. Using the Move tool choose a vertex of triangle P. Note how any changes of triangle P effect P'.

**CONJECTURE 1**

1. Make a conjecture about the congruence of triangle P and triangle P' when corresponding sides and angles exist.
2. Identify any congruence postulates that can be used to prove that the transformation is an isometric, or that a transformation preserves length and angle measure.

**CONSTRUCT 2 (REFLECTION OF A GEOMETRIC FIGURE OVER A LINE)**




1. Using the Polygon tool, draw a triangle. Right click on the triangle and rename the triangle S. Make sure to turn on the show label feature.
2. Select the New Point tool. Create two new points D and E outside of triangle S. Choose the Line Through Two Points Tool and choose points D and E. Right click on the line and rename it Q.

3. Using the Reflect Object about Line Tool choose triangle S and line Q. Name the reflection of the triangle S'.

**INVESTIGATE 2**

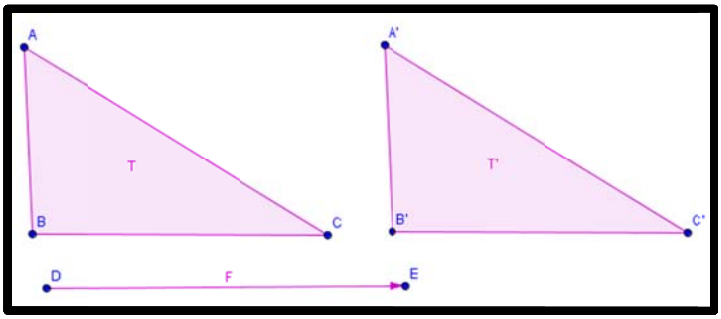
1. Measure the lengths of segments AB, AC, BC, A'B', A'C', B',C'. Click on each segment, length will appear highlighted on left. For lengths to appear on the image select the Distance or Length tool and select the segments. Name the sides of triangle S that are congruent to the sides of triangle S'.
2. Select the Angle tool. Measure angles ABC, BCA, CAB, A'B'C', B'C'A, C'A'B'. Name the angles of triangle S that are congruent to the angles of triangle S'.
3. Choose the Distance or Length tool. Select vertices of triangle S or S' and line Q. Measure all of the distances of the vertices to the line of reflection. Connect the vertices of triangle S and triangle S' that have the same lengths to line Q. Hint: There will be three line segments.
4. Using the Move tool choose a vertex of triangle S. Note how any changes of triangle S effect S'.

**CONJECTURE 2**

1. Write a conjecture about a reflection being isometric, or that the transformation preserves length and angle measure.
2. Write a conjecture about how each of the three segments, found by joining the vertices on triangle S to triangle S' over the point of reflection, are perpendicular bisectors to the line of reflection. Explain with the construction above.

**CONSTRUCT 3 (TRANSLATION OF A GEOMETRIC FIGURE)**

1. Using the Polygon tool, draw a triangle with vertices A,B, and C. Right click and rename the triangle T.
2. Using the New Point tool create points D and E. Choose the Vector between Two Points tool and points D and E to create a vector. Label the vector F by right clicking on the vector and renaming it.




3. Select the Translate Object by Vector tool. Choose triangle T and vector F. Label the transformed triangle T'.

**INVESTIGATE 3**

1. Using the Distance or Length tool measure the length of segments AA', BB', and CC'. How do the lengths of the segments relate to each other?
2. Measure the lengths of segments AB, AC, BC, A'B', A'C', B',C'. Click on each segment, length will appear highlighted on left. For lengths to appear on the image select the Distance or Length tool and select the segments. Name the sides of triangle T that are congruent to the sides of triangle T'.
3. Select the Angle tool. Click on the triangle T. Measure angles ABC, BCA, CAB, A'B'C', B'C'A, C'A'B'. Name the angles of triangle T that are congruent to the angles of triangle T'.
4. Using the Move tool choose a vertex of triangle T. Note how any changes of triangle T affect T'.

**CONJECTURE 3**

1. Write a conjecture relating the lengths of segments AA', BB', and CC' to the translation vector F. Note any changes that occur when you move the vertices of triangle T.
2. Write a conjecture about a translation being an isometric transformation. Use the constructions from the investigation to explain congruence.

# Geometric Constructions
# Constructing Equilateral and Isosceles Triangles

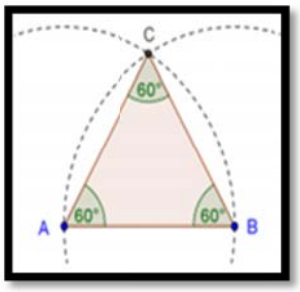
***Monica Doyle***

CCSS: Geometry – Congruence (G-CO - 12)

**CCSS: Geometric Constructions (G-CO -12)**

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing
perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line

**CONSTRUCT 1**




1. Select "Segment Between Two Points" tool and draw the segment AB.
2. Select "Circle with Center through Point" and draw the circle by clicking center point A first, then vertex B. Choose the mode "move" by selecting the arrow from the toolbar, then drag the points A and B to make sure the circle is connected to them.
3. Now using the same tool, draw a circle with center B through A.
4. Intersect both circles to get point C by selecting "Point-Intersect two objects" tool then clicking on the two circles. Notice that it gives you both points C and D. To hide point D, right click on the point and click "show object."
5. Use the "Polygon" tool to construct a polygon ABC in counterclockwise direction.

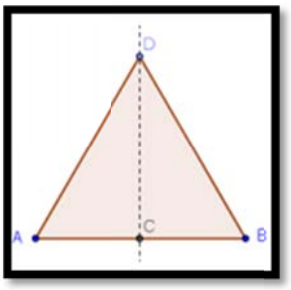
**INVESTIGATE**

- Measure the lengths of the segments AB, BC, and AC. What do you notice about them? (Click on "Tools"- "Measurement Tools"-"Distance or Length" then click on the segments you would like to measure. Notice that the measurements are also listed on the left side of the screen.)
- Measure the interior angles of the triangle by clicking on the "Angle" tool and the two segments that form the angle. What is unique about them?

**CONJECTURE – Do you think the following conjectures are true or false? Use the constructions above along with the "drag test" to see if you can find a counter-example.** (Use the drag test by selecting the "Move" tool then "dragging" any points on the object to see what happens to the triangle).

**Conjecture 1:** If all three sides of a triangle are *equal,* then all of the interior angles are congruent.
**Conjecture 2:** If a triangle is an equilateral triangle, then each interior angle is always 60 degrees. Is this true? Explain.

**CONSTRUCT 2 – Open a New Window** ----------




1. Select "Segment Between Two Points" tool and draw a segment AB.
2. Find the midpoint of segment AB by using the "Point-Midpoint or Center tool" and label this point C (right click on the point to label).
3. Select "Line-Perpendicular Line" to construct a perpendicular line to AB through point C.
4. Select "Point on Object" and place a new point, D, anywhere on the perpendicular line (preferably above segment AB).
5. Select "Segment Between Two Points" and construct segments AD and BD.

**INVESTIGATE**

- Measure the lengths of segments AB, BD, and AD. Use the "Move" tool to move point D up and down the perpendicular line.What do you notice about the lengths and their relationship?
- Can you come up with a different way to construct an isoceles triangle ? Write down the steps you use and why your construction works.

**CONJECTURE – Do you think the following conjectures are true or false? Use the constructions above along with the "drag test" to see if you can find a counter-example.** (Use the drag test by selecting the "Move" tool then "dragging" any points on the object to see what happens to the triangle).

**Conjecture 1:** If two sides of a triangle are equal in length, then there are two congruent angle in the triangle. If true, in general which two angles are these?
**Conjecture 2:** In an Isosceles triangle, all three angles must be acute (less than 90 degress). Explain.
**Conjecture 3:** If a triangle is Isoceles, then the base angles are congruent. Explain.

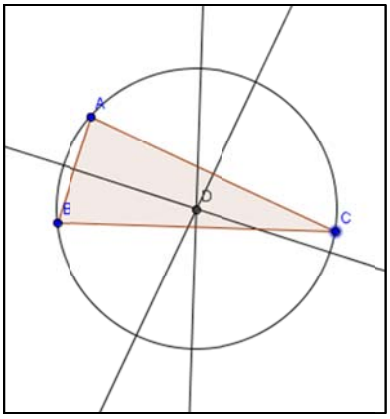
# Inscribed and Circumscribed Circles

***Laughlin Kane***

CCSS: Geometry – Circles (G-C - 3)

CCSS. GEOMETRY. G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

**Construct 1:**




1. Using the polygon tool, draw a triangle. Label the vertices A, B, and C.

2. Using the circle through three points tool, draw a circle using the three vertices as points. This is the "circumscribed circle" (or "circumcircle") of the triangle, since it passes through each of the polygon's vertices.

3. Select the perpendicular bisector tool. Draw the bisector of side AB, BC, and CA. Find where the lines cross each other. This is their point of intersection. Label it D.

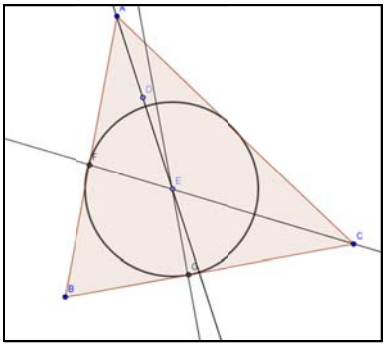
**Investigate 1:**

1. Draw another circle (using the center through point tool) with D as the center and any other point on the original circle as the point.
2. What do you notice about the new circle when compared to the original circle? So what is the center of this circle?
3. Select the arrow or "move" tool. Click on points A, B, and C and drag them around. If the circumcenter is point D (the intersection of the 3 perpendicular bisectors), then when is the circumcenter inside the triangle? When is it outside?
4. *Try* to repeat the above steps using other polygons, both regular and non-regular. Start with quadrilaterals then move onto polygons with 5, 6, 7, etc. sides. What do you notice?

**Conjecture 1:**

1. Make a conjecture about which polygons can and cannot be circumscribed by a circle. Test it.

**Construct 2:**



1. Using the polygon tool, draw a triangle. Label its three vertices A, B, and C.
2. Select the angle bisector tool. Draw the bisectors of two of the angles (say angle CAB and ACB). Find their point of intersection and label it D.
3. Using the perpendicular line tool, construct a perpendicular line from point D to one side of the triangle, say side BC.
4. Label the point where the perpendicular line crosses BC point G. This is the point of intersection of side BC and the perpendicular line.
5. Using the circle tool, draw a circle (use the circle with center through point tool) using D as the center and point G, such that the entire circle is within the triangle. This is an "inscribed circle," since it is the largest circle contained in the triangle and it touches all three sides.

**Investigate 2:**

1. *Try* to repeat the above steps using other polygons, both regular and non-regular. Start with quadrilaterals then move onto polygons with 5, 6, 7, etc. sides. What do you notice?

**Conjecture 2:**

1. Make a conjecture about which polygons can and cannot have an inscribed circle within them. Test it.

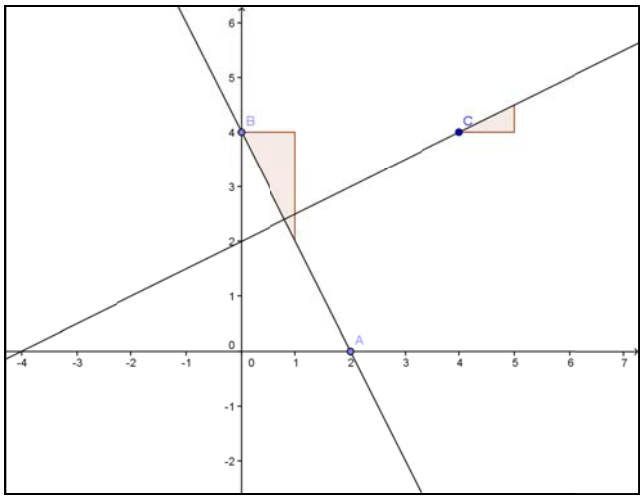
# Slope
# Discovering Slope
## *Colleen Hannan*
CCSS: Geometry – Expressing Geometric Properties with Equations (G-GPE - 5)

**CCSS**: (G-GPE – 5) Expressing Geometric Properties with Equations

**Use coordinates to prove simple geometric theorems algebraically**

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (g.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.)

**Construct 1**: Perpendicular Lines




**Goal**: Construct line AB through the points A(2,0) and B(0,4) and a line perpendicular to line AB through the point C(4,4).

1. Plot point (0,4) and (2,0)
    a. Turn on axes.
        i. View→Axes
    b. Click New Point box.
2. Create the points A(2,0), B(0,4), and C(4,4) on the graph by clicking where the point is located on the graph.
    a. The coordinates will appear next to the letter in the bar on the left side of the screen.
3. Create a line through the points A and B.
    a. Click "Line through 2 points" Tool.
    b. Select points A and B to create the line.

4. Create a line perpendicular to line AB and containing the point (4,4).
    a. Tools→Special Line Tools→Perpendicular Line
        i. Select point C and line AB to create the perpendicular line.
5. Find slope of line AB using the slope tool.
    a. Type: Slope[<Line>] into the Input bar located in the bottom of the window. Or…
    b. Then click line AB to find slope for line AB.
    c. Repeat to find slope of the perpendicular line.

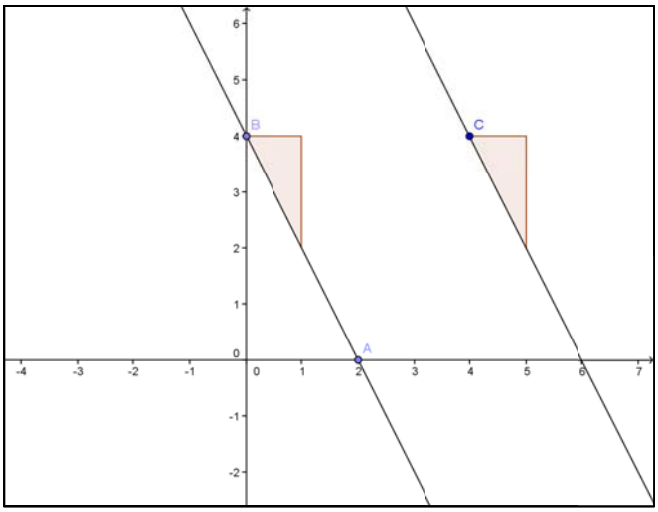
**Investigate 1**:

1. Have students work in pairs to create their own perpendicular lines and then find the slope.

**Conjecture 1**:

1. Write a conjecture about the relationship between the slopes of perpendicular lines. Use the data from the class and partner activity to support your answer.
    (Perpendicular lines slope is the inverse reciprocal!
    Ex: m=a then perpendicular line will have m=-1/a )

**Construct 2**: Parallel Lines




**Goal**: Construct line AB through the points A(2,0) and B(0,4) and a line parallel to line AB through the point C(4,4).

1. Repeat steps 1-3 from Construct 1.
2. Create a line parallel to line AB and containing the point (4,4).
    a. Tools→Special Line Tools→Parallel Line
        i. Select point C and line AB to create the parallel line.
3. Find slope of line AB using the slope tool.

a. Or Type: Slope[<Line>] into the Input bar located in the bottom of the window.
b. Then click line AB to calculate slope for line AB.
c. Repeat to find slope of the parallel line.

**Investigate 2**:

1. Have students work in pairs to create their own parallel lines and then find the slope.

**Conjecture 2**:

1. Write a conjecture about the relationship between the slopes of parallel lines. Use the data from the class and partner activity to support your answer.
   (Parallel Lines have the SAME slope!)

# GeoGebra Resources

**GeoGebra Homepage --- www.geogebra.org**

Includes downloads, teaching materials and a discussion board.